Course Code	Course Name	Credits
26PH504	ADVANCED MATHEMATICAL PHYSICS	04

Course Objectives

- Introduce foundational mathematical tools for quantum mechanics, including Hilbert spaces, Dirac notation, and operator algebra.
- Develop proficiency in solving partial differential equations (PDEs) using separation of variables and coordinate transformations.
- Build understanding of complex analysis techniques for contour integration and residue calculus.
- Equip students with skills in Fourier series, transforms, and Laplace transforms for signal processing and PDE solutions.
- Familiarize learners with special functions like Gamma, Beta, Legendre, and Hermite polynomials, including their properties and applications.

Learning Outcomes

After completion of the course, students will be able to:

- Explain Hilbert space concepts, apply Dirac notation to kets, bras, and operators, and derive uncertainty relations from commutators.
- Solve eigenvalue problems for Hermitian operators and perform unitary transformations in discrete and continuous bases.
- Classify and solve first- and second-order PDEs, including Laplace, Poisson, wave, and diffusion equations using separation of variables in various coordinates.
- Verify Cauchy-Riemann conditions, apply Cauchy's theorems, expand functions in Laurent series, and compute residues for definite integrals.

Unit 1 - Mathematical Tools of Vector and Matrix Spaces (12 Hrs.)

The Hilbert space and wave functions: Dirac Notation - Operators: Hermitian, Projection, commutator algebra, uncertainty relation between 2 operators, functions of operators, Eigenvalues and Eigenvectors of an operator, infinitesimal and finite unitary transformations. Representation in Discrete bases: Matrix representation of Kets, Bras and operators, change of bases and unitary transformations, Matrix representation of the Eigenvalues problems. Representation in Continuous bases: General treatment, position, momentum, connection representations.

Unit 2 – Partial Differential Equations (12 Hrs.)

First-Order equations: general PDEs, more than two independent variables - Second–Order equations: classes of PDEs, more than two independent variables - boundary conditions - Separation of variables: Cartesian coordinates, circular & cylindrical coordinates, spherical & polar coordinates - Laplace and Poisson equations - Wave equations: D'Alembert's solution - Diffusion PDE.

Unit 3 - Complex Analysis (12 Hrs.)

Cauchy - Riemann conditions - Cauchy's integral theorem - applications to multiply connected region - Cauchy's II integral theorem - derivatives of analytic complex function - singular points and their classification - Laurent series - Cauchy's residue theorem - calculation of residue at a point - evolution of definite integrals: (i) around the unit circle, (ii) around a semi-circular contour, and (iii) integral of the form.

Unit 4 – Fourier Series and Integral Transforms (12 Hrs.)

Fourier series: Dirichlet's condition - determination of coefficient - function having arbitrary period - Fourier series for square wave and half wave. Fourier Transform: FT of a time dependent function - some important theorems: Parseval's, linearity, derivatives, shifting of origin and convolution, use of FT in solving partial differential equation for heat conduction. Laplace transform: Theorems - inverse transform - solution to ordinary differential equations - solving equations for LCR circuit.

Unit 5 - Special Functions (12 Hrs.)

Gamma and Beta functions - properties and their basic relations - DE and series solution of Legendre and Hermite - their polynomial – Rodrigues' formula - generating function - recurrent relation - orthogonality relations.

Reference Books:

1. Arfken, Weber and Harris, "Mathematical Methods for Physicists", 7th Edition, Academic Press, 2013.
2. AK Mukhopadhyay, "Mathematical Methods for Engineers and Physicists", Wheeler Pub, New Delhi, 1998.
3. H.K Dass and Rama Verma, "Mathematical Physics", S. Chand publications, India 2011.

Websites and eLearning Sources:

1. https://nptel.ac.in/courses/115103036
2. https://youtu.be/FFPXm-tuOt8?si=069ZRFYSHXs3qzb0
3. https://youtu.be/da1rH0Hq62Q?si=NkJbDPmOUeyz2W5n

COs and Bloom's Taxonomy Mapping – 26PH504		
Course Outcomes	**On successful completion of this course, students will be able to**	**BTL**
CO1	Recall advanced mathematical tools such as operators, matrices, and Hilbert spaces.	K1, K2
CO2	Apply PDEs, Fourier transforms, and complex analysis in physical problems.	K3
CO3	Analyze physical systems using mathematical methods and transforms.	K4
CO4	Evaluate solutions of physical problems using advanced mathematical techniques.	K5
CO5	Develop mathematical models for complex physical systems.	K6

BTL (Bloom's Taxonomy Level) - K1 – Remembering, K2 – Understanding, K3- Applying, K4 – Analyse, K5- Evaluate and K6 - Create

Relationship Matrix – 26PH504													
Course Outcomes	**Programme Outcomes (POs)**						**Programme Specific Outcomes (PSOs)**						**Mean Score of Cos**
	PO1	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PSO1**	**PSO2**	**PSO3**	**PSO4**	**PSO5**	**PSO6**	
CO1	3	2	1	1	1	1	3	2	1	1	1	1	1.58
CO2	3	3	2	2	1	1	2	3	2	2	2	1	2.00
CO3	3	3	3	2	1	1	2	3	3	2	2	2	2.25
CO4	3	3	3	3	2	1	2	3	3	3	2	2	2.42
CO5	3	3	3	3	2	2	3	3	3	3	3	2	2.75
												Total	2.20

Mean Score: 3- High, 2- Medium/Moderate, 1-Low